Deciphering Daniel Tosh's Credit Card Number

By Jeffrey A. Shaffer (written 9/9/2013)

On Wednesday, September 3rd, 2013, Daniel Tosh ended his show Tosh.O with a real-world brain teaser.

> "The last 8 digits of Daniel's credit card are 6812 7167. The first 8 digits consist of the numerals 2, 3, 5, 6, 7, 8, 9, 0. What is the maximum number of guesses it would take before you could live the good life?"

I asked some analytics students at the University of Cincinnati how they might go about solving this problem. One student quickly responded, "it's 8 factorial". Mathematically 8!, or 40,320, is the correct answer for the number of possible permutations of an 8 digit number using these 8 digits. However, with some understanding about the composition of credit card numbers it is possible to quickly narrow this down to a much more manageable list. As soon as I saw this on TV I immediately posted to Facebook:

> "I really hope he didn't use his actual credit card number because there aren't as many guesses as he probably thinks there are. I could script that pretty quick and narrow that right down."

The folks at Identity Finder quickly figured this out as well and posted on their blog (http://bit.ly/1fFsNiq). Using their software they quickly identified 472 possibilities and narrowed that list down to 144 likely candidates (although they miscalculated the possible permutations of Discover Card numbers as 129 when it's actually 108). So how was this done? How is it possible that 40,320 combinations yield only 144 possible permutations? The solution is not that complex. It just requires some understanding about how credit card numbers work.

Credit card numbers are issued using the ANSI Standard X4.13-1983. This standard gives us the primary rules for filtering down the number of possibilities. Let's create a quick number sieve using these rules.

Step 1:

The first digit of a credit card number indicates the system for the card.

> 3 is the first digit for Entertainment and Travel Cards and American Express
> 4 is the first digit for Visa Card
> 5 is the first digit for MasterCard
> 6 is the first digit for Discover Card

Just simply using this filter it quickly eliminates 2, 7, 8, 9 and 0 as being the first digit. In addition, American Express accounts are not 16 digit numbers and most of the Travel and Entertainment cards are obsolete (and often times had different numbering schemes too). Based on this we can eliminate digit 3 as well. At this point we only have 2 choices for the first digit, either 5 or 6, and then 7! possibilities for the rest of the numbers. We're now down to 10,080 permutations after step 1. Rather than reinventing the wheel, I used this permutation macro at http://bit.ly/Np1sB to generate the 10,080 combinations.

Step 2:

The next step is where most of the magic happens. In 1954 an IBM scientist by the name of Hans Peter Luhn filed a patent for an algorithm to validate identification numbers. This algorithm is the foundation for most credit card numbers today. It allows a merchant, for example a merchant website, to quickly validate if a credit card is valid or not. Note that the last number of the credit card number is the checksum digit for the algorithm. Here is the algorithm as published on Wikipedia:

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).
2. Take the sum of all the digits.
3. If the total modulo 10 is equal to 0 (if the total ends in zero) then the number is valid according to the Luhn formula; else it is not valid.

The Luhn algorithm enables the merchant to instantly know if there is an incorrect digit in the credit card number and will also catch many of the transposed digits (2 digits being swapped). It does not indicate if the credit card was actually issued, but only if it's a valid number that could be issued.

Here's a quick illustration to show how this algorithm works with a 16 digit credit card number.

5	4	2	4	6	1	8	3	1	5	4	6	1	2	4	6	
X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	
10	4	4	4	12	1	16	3	2	5	8	6	2	2	8	6	
1+0	+4	+4	+4	+1+2	+1	+1+6	+3	+2	+5	+8	+6	+2	+2	+8	+6	**=66**

66 = 6 mod 10
After dividing by 10 the remainder = 6.
This is not a valid credit card number.

For this particular sequence of 15 numbers to be a valid credit card number there is only one possible answer for the checksum digit (the last digit of the credit card number). It has to be 0.

5	4	2	4	6	1	8	3	1	5	4	6	1	2	4	**0**	
X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	X 2	X 1	
10	4	4	4	12	1	16	3	2	5	8	6	2	2	8	0	
1+0	+4	+4	+4	+1+2	+1	+1+6	+3	+2	+5	+8	+6	+2	+2	+8	+0	**=60**

60 = 0 mod 10
After dividing by 10 the remainder = 0.
This is a valid credit card number.

Using the Luhn algorithm it is now possible to narrow down the valid credit card numbers from the 10,080 permutations. This step utilizes a Luhn algorithm macro in Excel to test the checksum. There are dozens of these available online, but I used this one at http://bit.ly/16f7H9k. Once the Luhn algorithm is applied there are only 720 permutations that are valid credit card account numbers.

Step 3:

The first 6 digits of a credit card account number is known as the Issuer Identification Number (formerly known as BIN). The issuer identification numbers are assigned to the banks and credit card companies. For example, 542432 is assigned to Fifth Third and 424631 is assigned to Chase Bank. Here is a simple lookup tool online that will return the issuer for any IIN at http://bit.ly/K3BeHh.

Now the average person won't have easy access to an updated IIN database, but even without this it's still possible to narrow the list even further. Using the basic issuer identification number ranges posted on Wikipedia (http://en.wikipedia.org/wiki/Bank_card_number) the list quickly narrows down to 144. There are 36 MasterCard possibilities and 108 Discover Card possibilities.

This list can be narrowed down further with some reasonable assumptions.

For example:

One of the issuing banks in the list of 144 credit card numbers is RAIFFEISENBANK A.S. in the Czech Republic, which is an unlikely candidate. However, another possible credit card number was issued by Fidelity Bank and another by Citigroup. These would be much more likely to be active accounts for someone living in the United States. Based on the number of good MasterCard numbers vs. the number of good Discover Card numbers it's likely that the actual credit card is a Discover Card.

End Result:

As demonstrated here, using some basic knowledge of how credit card numbers work and a few simple tools it is possible to narrow down the 40,320 permutations down to a very short list of possible credit card numbers and an even shorter list of likely candidates.

I have chosen not to publish the list of the 144 credit card numbers or the subset of likely candidates from that list, but unfortunately it would be very easy for anyone to do this. I hope that Daniel Tosh used a fake credit card number to begin with, but if it was a real credit card number then I sure hope he requested a new card.

There is a valuable lesson here. Always keep your credit card number secure.

Sincerely,

Jeffrey A. Shaffer